

On Holographic Dual of the Dyonic Reissner-Nordström Black Hole

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Abstract

It is shown that the hidden conformal symmetry, namely $SO(2,2) \sim SL(2,R)_L \times SL(2,R)_R$ symmetry, of the non-extremal dyonic Reissner-Nordström black hole can be probed by a charged massless scalar field at low frequencies. The existence of such hidden conformal symmetry suggests that the field theory holographically dual to the 4D Reissner-Nordström black hole indeed should be a 2D CFT. Although the associated AdS_3 structure does not explicitly appear in the near horizon geometry, the primary parameters of the dual CFT_2 can be exactly obtained without the necessity of embedding the 4D Reissner-Nordström black hole into 5D spacetime. The duality is further supported by comparing the absorption cross sections and real-time correlators obtained from both the CFT and the gravity sides.

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I. INTRODUCTION

Searching for the quantum gravity descriptions of black holes has a long story. The conceivable first explicit example is the calculation of the central charge of 2D CFT by analyzing the asymptotic symmetries of the asymptotically AdS_3 spacetimes, namely the $\text{AdS}_3/\text{CFT}_2$ correspondence [1]. Later, based on the ideas of the holographic principle [2, 3] and its first realization in string theory, i.e. $\text{AdS}_5/\text{CFT}_4$ correspondence [4–6], much success have been made along this direction especially for the recent progresses initiated by studying the quantum gravity descriptions of the Kerr black hole, i.e. the Kerr/CFT correspondence [7], together with its various applications and extensions [8–44]. In almost all the above cases, in order to apply the technique of the AdS/CFT duality, the background spacetime (in the extremal or near extremal limit) needs to contain some asymptotical or/and near horizon AdS structures. The Kerr/CFT correspondence is of no exception in the beginning, in which the near horizon $\text{AdS}_2 \times S^1$ geometry (with $SL(2, R)_R \times U(1)_L$ symmetry) of the (near) extremal Kerr black hole plays an essential role in obtaining the central charge of the dual 2D CFT.

The S^1 or $U(1)$ bundle of the Kerr black hole comes from the rotation, thus for non-rotating black holes such as the Reissner-Nordström (RN) black hole which only has the near horizon (near) extremal AdS_2 structure, the Kerr/CFT approach does not work directly. One possible way is to uplift the 4D RN black hole into 5D spacetime and let the extra dimension be a compactified S^1 circle (with radius ℓ), then the left hand central charge $c_L = 6Q^3/\ell$ of the dual CFT can be calculated [15–17]. Another way is to reduce the 4D RN black hole into two dimensions and study the dynamics of the stress tensor and current of the 2D effective theory, which results, with a suitable choice of an undetermined normalization factor, the right hand central charge $c_R = 6Q^2$ of the dual CFT [18]. The first picture shows that the near horizon (near) extremal uplifted 5D RN black hole has the (warped) $\text{AdS}_3/\text{CFT}_2$ description. While the second picture indicates that the near horizon (near) extremal 4D RN black hole has the $\text{AdS}_2/\text{CFT}_1$ description. Hence it is suggested in [17, 18] that the 4D and 5D RN black holes may serve as a possible example to study the relationship between the $\text{AdS}_3/\text{CFT}_2$ and $\text{AdS}_2/\text{CFT}_1$ dualities. On the other hand, the Kerr/CFT correspondence is further conjectured to work even in the generic non-extremal case based on the fact that the hidden conformal symmetry of the Kerr black hole can be probed by the external fields in the low frequency limit [45]. In this method, the $U(1)$ bundle also plays a vital important part in constructing the Casimir operator of the $SL(2, R)_L \times SL(2, R)_R$ Lie algebra. We soon show that the hidden conformal symmetry can be found also for the 5D RN black hole, which indicates that the generic non-extremal 5D RN black hole is dual to a 2D CFT [46]. For other related works with the hidden conformal symmetry, see [47–57].

However, whether the 4D RN black hole is dual the 2D or 1D CFTs remains an interesting problem. In the present paper we find that, even for the 4D non-extremal RN black hole, there should also exist a dual CFT_2 description, without the necessity of uplifting it into higher dimensions. What is more, the dyonic RN solution does not have a trivial embedding into five dimensional spacetime, so it is more natural to study the CFT dual directly in 4 dimensions. Although the techniques we used is similar to those used in the Kerr black hole case, the key point is that the $U(1)$ bundle of the 4D RN black hole can be probed by a charged scalar field but not just a neutral one. This is not surprising since the $U(1)$ symmetry is actually from the gauge symmetry of the background electromagnetic field. In practice, the gauge potential provides, via the coupling with a charged scalar field, the $U(1)$ fibration over the AdS_2 base manifold to form an AdS_3 structure, as we expect. Then the

generic 4D dyonic RN black hole is conjectured to be dual to a 2D CFT with $T_L = \frac{(r_+^2 + r_-^2)\ell}{4\pi Q(Q^2 + P^2)}$ and $T_R = \frac{(r_+^2 - r_-^2)\ell}{4\pi Q(Q^2 + P^2)}$, and $c_L = c_R = \frac{6Q(Q^2 + P^2)}{\ell}$. Besides the matching of the microscopic and macroscopic entropies

$$S_{CFT} = \frac{\pi^2}{3}(c_L T_L + c_R T_R) = \pi r_+^2 = S_{BH},$$

we also find that, the absorption cross section of the scalar field calculated from the gravity side matches with those of its dual operators in the 2D CFT exactly, and the real time correlators obtained from the gravity and the CFT sides are in good agreement up to some normalization factors depending on the charge of the probe field, which give further supports to the $\text{AdS}_3/\text{CFT}_2$ picture of the 4D RN black hole. Based on this work, we would like to point out that a dimensional reduction in the gravity side, namely from uplifting 4D RN black hole to its 5D counterpart, can not provide connection between the dual CFT_2 and CFT_1 . Actually, no matter for 4D RN or 5D RN black holes, their corresponding CFTs should be two dimensional.

The outline of this paper is as follows. We first review the basic properties of the dyonic RN black hole and then analyze the scattering process of a charged scalar field propagating in this background in Section II. In Section III, we illustrate how the hidden conformal symmetry of the 4D RN black hole is probed by a charged massless scalar field in the low frequency limit, and consequently, how the dual CFT_2 description appears. In Sections IV and V, we further verify the $\text{AdS}_3/\text{CFT}_2$ picture by comparing the absorption cross sections and the real time correlators calculated from both the gravity and the CFT_2 sides. Then we give the conclusion in Section VI. In Appendix A, we list the symmetry and the Casimir operator in the AdS_3 spacetime.

II. CHARGED SCALAR FIELD IN THE DYONIC RN BLACK HOLE

We begin by studying a charged probe scalar field propagating in the dyonic RN black hole background. The dyonic RN black holes are the spherically symmetric charged solutions of the four-dimensional Einstein-Maxwell theory

$$I_4 = \frac{1}{16\pi} \int d^4x \sqrt{-g} (R - F_{[2]}^2), \quad (1)$$

it is characterized by three parameters: one is the mass M and the other two are electric charge Q and magnetic charge P respectively,

$$\begin{aligned} ds^2 &= -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2, \\ A_{[1]} &= -\frac{Q}{r}dt + P \cos \theta d\phi, \end{aligned} \quad (2)$$

where

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2 + P^2}{r^2}. \quad (3)$$

The corresponding outer/inner horizon radius r_{\pm} , chemical potential Φ_H , Hawking temperature T_H and black hole entropy S_{BH} are

$$\begin{aligned} r_{\pm} &= M \pm \sqrt{M^2 - Q^2 - P^2}, \\ \Phi_H &= \frac{Q}{r_+}, \\ T_H &= \frac{1}{4\pi} \frac{r_+ - r_-}{r_+^2}, \\ S_{BH} &= \frac{A_+}{4} = \pi r_+^2. \end{aligned} \quad (4)$$

For a probe charged massive scalar field Φ of mass μ and electric charge q , which is minimally coupled to the dyonic RN black hole background, its corresponding Klein-Gordon (KG) equation

$$(\nabla_{\alpha} - iqA_{\alpha})(\nabla^{\alpha} - iqA^{\alpha})\Phi + \mu^2\Phi = 0, \quad (5)$$

can be simplified by assuming the following mode expansion of the scalar field

$$\Phi(t, r, \theta, \phi) = e^{-i\omega t + im\phi} S(\theta) R(r), \quad (6)$$

and, accordingly the KG equation reduces to two decoupled radial and angular equations by separation of variables:

$$\partial_r(\Delta \partial_r R) + \left[\frac{(\omega r - qQ)^2 r^2}{\Delta} - \mu^2 r^2 - \lambda_l \right] R = 0, \quad (7)$$

$$\frac{1}{\sin \theta} \partial_{\theta}(\sin \theta \partial_{\theta} S_l) + \left[\lambda_l - \frac{(m - qP)^2}{\sin^2 \theta} \right] S_l = 0, \quad (8)$$

where $\Delta = r^2 f = (r - r_+)(r - r_-)$. For the massless scalar field, $\mu = 0$, the radial equation (7) can be further reformulated in the following symmetric form

$$\begin{aligned} \partial_r(\Delta \partial_r R) + \left[\frac{(r_+^2 \omega - qQr_+)^2}{(r - r_+)(r_+ - r_-)} - \frac{(r_-^2 \omega - qQr_-)^2}{(r - r_-)(r_+ - r_-)} \right. \\ \left. + (\omega r + 2\omega M - qQ)^2 - \omega^2(2Mr + Q^2 + P^2) \right] R = \lambda_l R. \end{aligned} \quad (9)$$

In order to reveal the 2D conformal symmetry in the radial part equation of motion (9), one should impose proper conditions on the probe scalar field (6) such that the potential terms in the second line of Eq.(9) can be neglected. These conditions are: (i) low frequency: $\omega M \ll 1$ (consequently $\omega Q \ll 1$ and $\omega P \ll 1$), (ii) small electric charge: $qQ \ll 1$ and $qP \ll 1$, (iii) near region: $\omega r \ll 1$. Note that the condition $qP \ll 1$ simplifies the angular equation in the way that the separation constant should take the standard value $\lambda_l = l(l+1)$, and the solutions for S_l are just the standard spherical harmonic functions.

III. HIDDEN CONFORMAL SYMMETRY

The reduced radial equation (9), after imposing the suitable conditions at the end of the Section II, is

$$\partial_r(\Delta\partial_r R) + \left[\frac{r_+^4(\omega + A_+q)^2}{(r - r_+)(r_+ - r_-)} - \frac{r_-^4(\omega + A_-q)^2}{(r - r_-)(r_+ - r_-)} \right] R = l(l+1)R, \quad (10)$$

where $A_\pm = -Q/r_\pm$ are the gauge potentials at the outer/inner horizons. Defining the covariant derivative operators as $D_\pm = \partial_t - iqA_\pm$, then we can have the relations $D_\pm \Phi = -i(\omega + qA_\pm)\Phi$. Consequently, we find that the reduced radial equation (10) matches with the Casimir operator of the $SL(2, R)_L \times SL(2, R)_R$ Lie algebra (A8) by imposing the following relations

$$\begin{aligned} -r_+^2 D_+ &= (r_+ - r_-) \left(\frac{T_L + T_R}{4\mathcal{A}} \partial_t - \frac{n_L + n_R}{4\pi\mathcal{A}} \partial_\chi \right), \\ -r_-^2 D_- &= (r_+ - r_-) \left(\frac{T_L - T_R}{4\mathcal{A}} \partial_t - \frac{n_L - n_R}{4\pi\mathcal{A}} \partial_\chi \right), \end{aligned} \quad (11)$$

where \mathcal{A} is defined in (A7). Here a suitable sign is chosen to ensure the temperatures to be positive. In the above identifications, we have introduced an operator ∂_χ , which can be naturally considered to act on the “internal” $U(1)$ symmetry space of the probe electrically charged scalar field and its eigenvalue gives the electric charge of the probe field such as $\partial_\chi \Phi = i\ell q \Phi$, where the parameter ℓ is, in general, an arbitrary normalization factor and it should not affect entropy of the dual 2D CFT. Thus, the covariant derivatives can be rewritten as $D_\pm = \partial_t - (A_\pm/\ell)\partial_\chi = \partial_t + (Q/\ell r_\pm)\partial_\chi$ and the relations (11) decompose into four algebraic equations

$$\begin{aligned} -r_+^2 &= \frac{(r_+ - r_-)(T_L + T_R)}{4\mathcal{A}}, & -r_+^2 A_+ &= \ell \frac{(r_+ - r_-)(n_L + n_R)}{4\pi\mathcal{A}}, \\ -r_-^2 &= \frac{(r_+ - r_-)(T_L - T_R)}{4\mathcal{A}}, & -r_-^2 A_- &= \ell \frac{(r_+ - r_-)(n_L - n_R)}{4\pi\mathcal{A}}. \end{aligned} \quad (12)$$

The temperatures of the left hand- and right hand- CFTs, i.e. T_L and T_R , then can be straightforwardly computed

$$T_L = \frac{(r_+^2 + r_-^2)\ell}{4\pi Q r_+ r_-} = \frac{(r_+ + r_-)M\ell - (Q^2 + P^2)\ell}{2\pi Q(Q^2 + P^2)}, \quad (13)$$

$$T_R = \frac{(r_+^2 - r_-^2)\ell}{4\pi Q r_+ r_-} = \frac{(r_+ - r_-)M\ell}{2\pi Q(Q^2 + P^2)}, \quad (14)$$

as well as the two other quantities n_L and n_R

$$n_L = -\frac{r_+ + r_-}{4r_+ r_-} = -\frac{r_+ + r_-}{4(Q^2 + P^2)}, \quad n_R = -\frac{r_+ - r_-}{4r_+ r_-} = -\frac{r_+ - r_-}{4(Q^2 + P^2)}. \quad (15)$$

In addition, the central charges of the dual CFT corresponding to the dyonic RN black hole have been studied both in the 5D uplifted picture and the 2D reduced picture via analyzing the asymptotical symmetry of the near horizon (near) extremal geometry [15–18]. As has been discussed in [46] that in both of the two approaches, there is an undetermined free parameter ℓ' — may not be the same with the parameter ℓ at the moment — appearing in the temperatures and central charges of the dual CFTs. The generic expression for the central charges are

$$c_L = c_R = \frac{6Q(Q^2 + P^2)}{\ell'}. \quad (16)$$

Note that the value of ℓ' in the 5D picture labels the radius of extra dimensional circle while ℓ in the 4D picture also describes the size of an “internal” circle. With the relation $\partial_\chi \Phi = i\ell q \Phi$, it is clear to see that Eq.(10) is exactly the same as the radial part equation for a neutral probe scalar field scattering in the 5D uplifted RN black hole background at low frequencies [46]¹. Therefore, we should identify $\ell' = \ell$ and then the CFT microscopic entropy characterized by the Cardy formula agrees with the black hole macroscopic area entropy

$$S_{\text{CFT}} = \frac{\pi^2}{3}(c_L T_L + c_R T_R) = \pi r_+^2 = S_{\text{BH}}. \quad (17)$$

Like the Kerr black hole and the 5D RN black hole, the hidden conformal symmetry, i.e. the $SL(2, R)_L \times SL(2, R)_R$ symmetry of the generic non-extremal dyonic RN black hole cannot be detected globally in the solution space of Eq.(10), it will break from $SL(2, R)_L \times SL(2, R)_R$ into $U(1)_L \times U(1)_R$ due to the periodic identification of the internal $U(1)$ symmetry

$$\chi \sim \chi + 2\pi. \quad (18)$$

¹ This fact actually indicates that the 4D and 5D RN black holes should dual to the same 2D CFT.

Accordingly, the generators of the $SL(2, R)_L \times SL(2, R)_R$ Lie algebra listed in Appendix A transform as

$$w^+ \sim e^{4\pi^2 T_R} w^+, \quad w^- \sim e^{4\pi^2 T_L} w^-, \quad y \sim e^{2\pi^2 (T_L + T_R)} y. \quad (19)$$

IV. SCATTERING OF THE PROBE SCALAR FIELD

In addition to the matching of CFT and black hole entropies, the dyonic RN/CFT₂ correspondence can be further checked by studying the scattering of the probe charged scalar field in the RN black hole background. We will see that the absorption cross section of the scalar field is in agreement with the two point function of its corresponding operators in the dual 2D CFT.

From the gravity side², the near region KG equation (10) can be easily solved by introducing a new variable z

$$z = \frac{r - r_+}{r - r_-}, \quad (20)$$

and the general solution includes both ingoing and outgoing modes

$$\begin{aligned} R^{(\text{in})} &= z^{-i\gamma} (1 - z)^{l+1} F(a, b; c; z), \\ R^{(\text{out})} &= z^{i\gamma} (1 - z)^{l+1} F(a^*, b^*; c^*; z), \end{aligned} \quad (21)$$

in which $F(a, b; c; z)$ is the hypergeometric function and all the coefficients are given by

$$\begin{aligned} \gamma &= \frac{(\omega r_+ - qQ)r_+}{r_+ - r_-}, \\ a &= 1 + l - i \frac{\omega(r_+^2 + r_-^2) - qQ(r_+ + r_-)}{r_+ - r_-}, \\ b &= 1 + l - i [\omega(r_+ + r_-) - qQ], \\ c &= 1 - i2\gamma. \end{aligned} \quad (22)$$

An useful relation among these coefficients is

$$c - a - b = -2l - 1. \quad (23)$$

² Note that even though the calculation of scattering amplitudes does not relate to the geometric quantities in the gravity side, directly, it does extract the information of the geometric background since the probe field is coupled to the gravity.

In the matching region $r \gg M$ and $r \ll 1/\omega$ (which is equivalent to take the limits $z \rightarrow 1$ and $1 - z \rightarrow r^{-1}$), the ingoing mode asymptotically behaves as

$$R^{(\text{in})}(r \gg M) \sim A r^l + B r^{-l-1}, \quad (24)$$

where two coefficients A and B are

$$A = \frac{\Gamma(c)\Gamma(2l+1)}{\Gamma(a)\Gamma(b)}, \quad B = \frac{\Gamma(c)\Gamma(-2l-1)}{\Gamma(c-a)\Gamma(c-b)}. \quad (25)$$

One more additional information can be obtained from (24) is the conformal weights of the scalar field

$$h_L = h_R = l + 1. \quad (26)$$

Hence, the coefficients a and b can be expressed in terms of conformal weights (real part) and two parameters $\tilde{\omega}_L$ and $\tilde{\omega}_R$ (imaginary part)

$$a = h_R - i \frac{\tilde{\omega}_R}{2\pi T_R}, \quad b = h_L - i \frac{\tilde{\omega}_L}{2\pi T_L}, \quad (27)$$

where $\tilde{\omega}_L, \tilde{\omega}_R$ are composed by three sets of the parameters: frequencies (ω_L, ω_R) , charges (q_L, q_R) and chemical potentials (μ_L, μ_R)

$$\begin{aligned} \tilde{\omega}_L &= \omega_L - q_L \mu_L, & \omega_L &= \frac{\ell \omega (r_+ + r_-)(r_+^2 + r_-^2)}{2Q r_+ r_-}, & q_L &= q, & \mu_L &= \frac{\ell (r_+^2 + r_-^2)}{2r_+ r_-}, \\ \tilde{\omega}_R &= \omega_R - q_R \mu_R, & \omega_R &= \frac{\ell \omega (r_+ + r_-)(r_+^2 + r_-^2)}{2Q r_+ r_-}, & q_R &= q, & \mu_R &= \frac{\ell (r_+ + r_-)^2}{2r_+ r_-}. \end{aligned} \quad (28)$$

The essential part of the absorption cross section of the charged scalar field can be read out directly from the coefficient A in Eq.(24), namely

$$P_{\text{abs}} \sim |A|^{-2} \sim \sinh(2\pi\gamma) |\Gamma(a)|^2 |\Gamma(b)|^2. \quad (29)$$

In order to compare the absorption cross section (29) with the two-point function of the operator dual to the probe scalar field, one needs to identify the conjugate charges, δE_L and δE_R , defined by

$$\delta S_{CFT} = \frac{\delta E_L}{T_L} + \frac{\delta E_R}{T_R}, \quad (30)$$

from the first law of black hole thermodynamics

$$\delta S_{BH} = \frac{1}{T_H} \delta M - \frac{\Phi_H}{T_H} \delta Q. \quad (31)$$

Here the magnetic charge is fixed in the first law, i.e. $\delta P = 0$, since the probe scalar field carries only the electric charge, therefore, it can not probe the the background magnetic charge and hence cannot take the magnetic charge away from the black hole. Finally one can get the conjugate charges via $\delta S_{CFT} = \delta S_{BH}$ and the solution is

$$\begin{aligned}\delta E_L &= \frac{2\ell(2M^2 - Q^2 - P^2)M}{Q(Q^2 + P^2)}\delta M - \frac{\ell(2M^2 - Q^2 - P^2)}{Q^2 + P^2}\delta Q, \\ \delta E_R &= \frac{2\ell(2M^2 - Q^2 - P^2)M}{Q(Q^2 + P^2)}\delta M - \frac{2\ell M^2}{Q^2 + P^2}\delta Q.\end{aligned}\quad (32)$$

These conjugate charges are actually identical with ω_L and ω_R by identifying $\delta M = \omega$ and $\delta Q = q$, namely

$$\tilde{\omega}_L = \delta E_L(\delta M = \omega, \delta Q = q), \quad \tilde{\omega}_R = \delta E_R(\delta M = \omega, \delta Q = q). \quad (33)$$

Moreover, one can also straightforwardly verify the following relation for the imaginary part of the coefficient c ,

$$2\pi\gamma = \frac{\tilde{\omega}_L}{2T_L} + \frac{\tilde{\omega}_R}{2T_R}. \quad (34)$$

Then the absorption cross section ultimately can be expressed as

$$P_{\text{abs}} \sim T_L^{2h_L-1} T_R^{2h_R-1} \sinh\left(\frac{\tilde{\omega}_L}{2T_L} + \frac{\tilde{\omega}_R}{2T_R}\right) \left| \Gamma\left(h_L + i\frac{\tilde{\omega}_L}{2\pi T_L}\right) \right|^2 \left| \Gamma\left(h_R + i\frac{\tilde{\omega}_R}{2\pi T_R}\right) \right|^2, \quad (35)$$

which has the same form of the finite temperature absorption cross section of an operator with the conformal weights (h_L, h_R) , frequencies (ω_L, ω_R) , electric charges (q_L, q_R) and chemical potentials (μ_L, μ_R) in the dual 2D CFT with the temperatures (T_L, T_R) .

V. REAL TIME CORRELATOR

Furthermore, since the real time correlator or the retarded Green's function are of physical causal meaning, it is of importance to calculate them in the AdS/CFT correspondence [58]. For the Kerr black hole, this has been studied in [41, 42, 55], so it is very interesting to calculate the real time correlator from the perspective of the RN/CFT₂ duality.

From the gravity side, note that the asymptotic behavior of the ingoing scalar field (24) indicates that two coefficients play different roles: A as the source and B as the response, thus the two-point retarded correlator is simply [41]

$$G_R \sim \frac{B}{A} = \frac{\Gamma(-2l-1)}{\Gamma(2l+1)} \frac{\Gamma(a)\Gamma(b)}{\Gamma(c-a)\Gamma(c-b)}. \quad (36)$$

Together with the identity (23), we can easily check that retarded Green function is

$$G_R \sim \frac{\Gamma\left(h_L - i\frac{\tilde{\omega}_L}{2\pi T_L}\right) \Gamma\left(h_R - i\frac{\tilde{\omega}_R}{2\pi T_R}\right)}{\Gamma\left(1 - h_L - i\frac{\tilde{\omega}_L}{2\pi T_L}\right) \Gamma\left(1 - h_R - i\frac{\tilde{\omega}_R}{2\pi T_R}\right)}. \quad (37)$$

Using the relation $\Gamma(z)\Gamma(1-z) = \pi/\sin(\pi z)$ we have

$$G_R \sim \sin\left(\pi h_L + i\frac{\tilde{\omega}_L}{2T_L}\right) \sin\left(\pi h_R + i\frac{\tilde{\omega}_R}{2T_R}\right) \Gamma\left(h_L - i\frac{\tilde{\omega}_L}{2\pi T_L}\right) \Gamma\left(h_L + i\frac{\tilde{\omega}_L}{2\pi T_L}\right) \Gamma\left(h_R - i\frac{\tilde{\omega}_R}{2\pi T_R}\right) \Gamma\left(h_R + i\frac{\tilde{\omega}_R}{2\pi T_R}\right). \quad (38)$$

Moreover, since the conformal weights $h_L = h_R = l + 1$ are integers so

$$\sin\left(\pi h_L + i\frac{\tilde{\omega}_L}{2T_L}\right) \sin\left(\pi h_R + i\frac{\tilde{\omega}_R}{2T_R}\right) = (-)^{h_L+h_R} \sin\left(i\frac{\omega_L - q_L\mu_L}{2T_L}\right) \sin\left(i\frac{\omega_R - q_R\mu_R}{2T_R}\right). \quad (39)$$

From the 2D CFT side, the Euclidean correlator, in terms of the Euclidean frequencies $\omega_{EL} = i\omega_L$ and $\omega_{ER} = i\omega_R$, is

$$G_E(\omega_{EL}, \omega_{ER}) \sim T_L^{2h_L-1} T_R^{2h_R-1} e^{i\frac{\tilde{\omega}_{EL}}{2T_L}} e^{i\frac{\tilde{\omega}_{ER}}{2T_R}} \Gamma\left(h_L - \frac{\tilde{\omega}_{EL}}{2\pi T_L}\right) \Gamma\left(h_L + \frac{\tilde{\omega}_{EL}}{2\pi T_L}\right) \Gamma\left(h_R - \frac{\tilde{\omega}_{ER}}{2\pi T_R}\right) \Gamma\left(h_R + \frac{\tilde{\omega}_{ER}}{2\pi T_R}\right), \quad (40)$$

where

$$\tilde{\omega}_{EL} = \omega_{EL} - iq_L\mu_L, \quad \tilde{\omega}_{ER} = \omega_{ER} - iq_R\mu_R. \quad (41)$$

The retarded Green function $G_R(\omega_L, \omega_R)$ is analytic on the upper half of the complex $\omega_{L,R}$ -plane and it is related to the Euclidean correlator by

$$G_E(\omega_{EL}, \omega_{ER}) = G_R(i\omega_L, i\omega_R), \quad \omega_{EL}, \omega_{ER} > 0, \quad (42)$$

and the Euclidean frequencies ω_{EL} and ω_{ER} should take discrete values of the Matsubara frequencies at finite temperature

$$\omega_{EL} = 2\pi m_L T_L, \quad \omega_{ER} = 2\pi m_R T_R, \quad (43)$$

in which m_L, m_R are integers for bosons and half integers for fermions. At these frequencies, the retarded Green function matches well with the gravity side computation (38) up to a normalization factor depending on the charges q_L and q_R , i.e. the electric charge of the probe scalar field.

VI. CONCLUSION

According to the well-known property that the near horizon (near) extremal geometry of the 4D RN black hole only contains an AdS_2 structure, thus it is natural to believe that the associated holographic dual CFT should be one-dimensional based on the idea of the usual $\text{AdS}_D/\text{CFT}_{D-1}$ correspondence. The CFT_2 description is expected to appear only until the 4D RN black hole is embedded into five-dimensional spacetime where part of the $U(1)$ gauge potential in 4D is transformed to the Kaluza-Klein vector as off-diagonal components in the uplifted 5D metric, and then a (warped) AdS_3 geometry comes out in the near horizon (near) extremal limit. Naively, by combining the uplifted and reduced perspectives, the 4D RN black hole and its uplifted 5D counterpart seem to provide a testable example for studying CFT_1 from CFT_2 by a simple dimensional reduction.

However, in this paper, from the technique of probing the hidden conformal symmetry of black hole backgrounds via external fields, we suggest that even for the generic non-extremal 4D dyonic RN black hole, it should still dual to a 2D CFT once we correctly incorporate the contribution of the background gauge field. In practice, the $U(1)$ symmetry of the background electromagnetic field can be probed by an external charged scalar field, consequently, the hidden 2D conformal symmetry of the 4D RN black hole is revealed. In mathematical language, the AdS_3 structure of the 4D RN black hole nevertheless is encoded in the fiber bundle where the near horizon (near) extremal AdS_2 geometry serves as the base manifold while the $U(1)$ gauge field acts as the fiber. Hence a dimensional reduction of the spacetime does not change the full symmetry structure, but merely transforms part of the geometric information from the base manifold into the fiber and vice versa. We investigate all the technical details to read out the dual CFT_2 information of the dyonic RN black hole from a probe charged scalar field and show that the charged scalar field can reveal equivalent CFT_2 information probed by a neutral scalar field scattering in the 5D uplifted RN black hole. Our results indicate a dyonic RN/ CFT_2 correspondence and the approach can be further generalized to more generic charged black holes. An interesting example is the Kerr-Newman black hole we can explore, besides a recently well-studied angular momentum $\text{AdS}_3/\text{CFT}_2$ description (which we call it the J -picture), there should exist another dual CFT_2 description in which the $U(1)$ symmetry of the background electromagnetic field can be probed (which we call it the Q -picture). We would like to report the results in a

forthcoming paper [59].

Appendix A: Symmetry and Casimir Operator of AdS₃

In this Appendix, we summarized the basic properties of the AdS₃ space which are useful for our study in this paper. The metric of the AdS₃ space with radius L , in the Poincaré coordinates: (w^\pm, y) , is

$$ds_3^2 = \frac{L^2}{y^2}(dy^2 + dw^+ dw^-). \quad (\text{A1})$$

There are two sets of symmetry generators

$$H_1 = i\partial_+, \quad H_0 = i\left(w^+ \partial_+ + \frac{1}{2}y \partial_y\right), \quad H_{-1} = i\left((w^+)^2 \partial_+ + w^+ y \partial_y - y^2 \partial_-\right), \quad (\text{A2})$$

$$\bar{H}_1 = i\partial_-, \quad \bar{H}_0 = i\left(w^- \partial_- + \frac{1}{2}y \partial_y\right), \quad \bar{H}_{-1} = i\left((w^-)^2 \partial_- + w^- y \partial_y - y^2 \partial_+\right), \quad (\text{A3})$$

assembling two copies of the $SL(2, R)$ Lie algebra

$$[H_0, H_{\pm 1}] = \mp i H_{\pm 1}, \quad [H_{-1}, H_1] = -2i H_0. \quad (\text{A4})$$

Thus the corresponding Casimir operator is

$$\mathcal{H}^2 = \bar{\mathcal{H}}^2 = -H_0^2 + \frac{1}{2}(H_1 H_{-1} + H_{-1} H_1) = \frac{1}{4}(y^2 \partial_y^2 - y \partial_y) + y^2 \partial_+ \partial_-. \quad (\text{A5})$$

The Poincaré metric (A1) can be transformed to a black hole metric with coordinates (t, r, χ) via the following coordinate transformations

$$\begin{aligned} w^+ &= \sqrt{\frac{r-r_+}{r-r_-}} \exp(2\pi T_R \chi + 2n_R t), \\ w^- &= \sqrt{\frac{r-r_+}{r-r_-}} \exp(2\pi T_L \chi + 2n_L t), \\ y &= \sqrt{\frac{r_+ - r_-}{r-r_-}} \exp[\pi(T_R + T_L)\chi + (n_R + n_L)t], \end{aligned} \quad (\text{A6})$$

we can directly calculate all the $SL(2, R)$ generators in terms of black hole coordinates

$$\begin{aligned} H_1 &= i e^{-(2\pi T_R \chi + 2n_R t)} \left[\sqrt{\Delta} \partial_r + \frac{n_L(\delta_- + \delta_+) + n_R(\delta_- - \delta_+)}{4\pi\sqrt{\Delta}\mathcal{A}} \partial_\chi - \frac{T_L(\delta_- + \delta_+) + T_R(\delta_- - \delta_+)}{4\sqrt{\Delta}\mathcal{A}} \partial_t \right], \\ H_0 &= i \left[\frac{n_L}{2\pi\mathcal{A}} \partial_\chi - \frac{T_L}{2\mathcal{A}} \partial_t \right], \\ H_{-1} &= i e^{2\pi T_R \chi + 2n_R t} \left[-\sqrt{\Delta} \partial_r + \frac{n_L(\delta_- + \delta_+) + n_R(\delta_- - \delta_+)}{4\pi\sqrt{\Delta}\mathcal{A}} \partial_\chi - \frac{T_L(\delta_- + \delta_+) + T_R(\delta_- - \delta_+)}{4\sqrt{\Delta}\mathcal{A}} \partial_t \right], \end{aligned}$$

and

$$\begin{aligned}\bar{H}_1 &= ie^{-(2\pi T_L \chi + 2n_L t)} \left[\sqrt{\Delta} \partial_r - \frac{n_R(\delta_- + \delta_+) + n_L(\delta_- - \delta_+)}{4\pi\sqrt{\Delta}\mathcal{A}} \partial_\chi + \frac{T_R(\delta_- + \delta_+) + T_L(\delta_- - \delta_+)}{4\sqrt{\Delta}\mathcal{A}} \partial_t \right], \\ \bar{H}_0 &= i \left[-\frac{n_R}{2\pi\mathcal{A}} \partial_\chi - \frac{T_R}{2\mathcal{A}} \partial_t \right], \\ \bar{H}_{-1} &= ie^{2\pi T_L \chi + 2n_L t} \left[-\sqrt{\Delta} \partial_r - \frac{n_R(\delta_- + \delta_+) + n_L(\delta_- - \delta_+)}{4\pi\sqrt{\Delta}\mathcal{A}} \partial_\chi + \frac{T_R(\delta_- + \delta_+) + T_L(\delta_- - \delta_+)}{4\sqrt{\Delta}\mathcal{A}} \partial_t \right],\end{aligned}$$

where

$$\delta_\pm = r - r_\pm, \quad \mathcal{A} = T_R n_L - T_L n_R. \quad (\text{A7})$$

Finally the Casimir operator becomes

$$\mathcal{H}^2 = \partial_r \Delta \partial_r - \frac{r_+ - r_-}{r - r_+} \left(\frac{T_L + T_R}{4\mathcal{A}} \partial_t - \frac{n_L + n_R}{4\pi\mathcal{A}} \partial_\chi \right)^2 + \frac{r_+ - r_-}{r - r_-} \left(\frac{T_L - T_R}{4\mathcal{A}} \partial_t - \frac{n_L - n_R}{4\pi\mathcal{A}} \partial_\chi \right)^2. \quad (\text{A8})$$

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